# Coherence resonance in excitable electronic circuits in the presence of colored noise

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We give evidence of coherence resonance in an excitable electronic circuit whose dynamics obeys the FitzHugh-Nagumo model system, under the application of different noise sources, ranging from Gaussian white noise to colored $1/f^2$ noises. The resonance behavior can be significantly reinforced when experimental parameters are tuned in order to place the stable fixed point closer to the excitability threshold of spiking behavior, as well as when the time scales of the circuit are properly modified. A quantitative description of the effects of noise correlations in inducing the resonant behavior is provided.

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In nature, most systems have inherent noise, or they are subjected to the action of external noisy sources. In several cases, such noise acts as a disturbance to the system dynamics, yielding a more complicated and ambiguous behavior. In nonlinear systems, however, noise in some cases can give rise to counterintuitive and constructive effects, as, e.g., it can activate a resonant response of the system.

In particular, in threshold systems, noise may play a constructive role through stochastic resonance (SR) [1] or coherence resonance (CR) [2]. In SR, noise optimizes the system's response and enhances synchronization to a weak external signal [1] when the noise-controlled time scale is close to that of the external source [3–5]. In CR, pure noise alone can generate coherent motion.

CR was originally predicted in excitable systems (ESs) [2], and later also demonstrated in bistable [6] and chaotic systems [7,8]. Recently, it has been also shown theoretically that a mechanical time-delayed bistable system exhibits CR in the presence of noise, due to the interplay between the internal time scale imposed by the time delay and the external time scale imposed by the noise [9,10]. Furthermore, evidence of CR has been given in an electronic experiment based on a monovibrator circuit [11], and in an optical system consisting of a laser diode with feedback [12].

Only recently were resonant effects of colored noise approached. Precisely, SR in sensory neurons forced by $1/f$ and $1/f^2$ noise was demonstrated [13], and resonant behaviors induced by Ornstein-Uhlenbeck processes were reported in integrate-and-fire models [14], as well as in excitable units [15]. Furthermore, Ref. [16] claims that CR effects in excitable systems persist in the presence of correlations in the noise source.

In this paper we show CR in excitable electronic circuits, under the application of a class of colored noise, whose spectral distribution scales as $1/f^{\alpha}$, with $0 \leqslant \alpha \leqslant 2$ being a tunable real parameter. By varying $\alpha$ we quantitatively assess the effects of noise correlation in the resonant process, and compare the scenario with the case of a Gaussian white noise. In all cases, CR can be properly enhanced by experimentally modulating the distance to the excitation threshold, or the time scales of the system's dynamics.

A system is called excitable when it has a stable fixed point with a finite basin of attraction. When a finite perturbation allows one to overcome a given threshold value, the return to the fixed point occurs by executing a large excursion in phase space, thus generating a pulsing or spiking behavior in the time evolution of the system's variables. An ES can therefore be locally regarded as a potential well, where large enough perturbations can kick off the system from the well, inducing a spiking return to it along a pulsed excursion orbit [17]. In these systems, CR has been predicted and verified, when a noise forcing is considered. CR indeed is a phenomenon by means of which the coherence (or degree of regularity) of noise-induced pulses shows a maximum for a certain optimal noise amplitude. The coherence can be maximized by the interplay between two characteristic time scales, namely, the activation time $t_a$ and the excursion time $t_e$. The activation time is the time needed to excite the system from the stable fixed point; the excursion time is the time needed to return from the excited state to the fixed point. The interspike interval $I_{\mathrm{IS}}$ is the time duration between two consecutive firings and is given by the sum of the two characteristic times $I_{\mathrm{IS}}=t_a+t_e$. The role of noise is therefore twofold: in the stable region it modulates the escape time from the fixed point, and in the excitable region it modulates the return orbit.

The degree of coherence $C$ of such motion can be defined as follows:

$$C = \frac{\langle I_{\mathrm{IS}} \rangle}{\sigma(I_{\mathrm{IS}})}, \tag{1}$$

where $\langle I_{\mathrm{IS}} \rangle [\sigma(I_{\mathrm{IS}})]$ is the average (standard deviation) of the distribution of interspike intervals. The mechanism leading to CR can be understood as follows. The activation time decreases rapidly with increasing noise amplitude. Thus, for small noise, where $t_a \gg t_e$, $I_{\mathrm{IS}}$ is dominated by the activation time $I_{\mathrm{IS}} \approx t_a$, and the noise-induced $I_{\mathrm{IS}}$ fluctuations are relatively large. On the contrary, for large enough noise, the contribution of the activation time $t_a$ to $I_{\mathrm{IS}}$ is negligible, and the excursion time dominates ($I_{\mathrm{IS}} \approx t_e$). If the motion in the excited state is nearly uniform, $\langle t_e \rangle$ depends only weakly upon the noise amplitude, so that the $I_{\mathrm{IS}}$ fluctuations grow as the noise amplitude increases. In these conditions, CR appears if the threshold of excitation is small and the excursion time is large. The maximum of $C$ corresponds to a large enough noise intensity to determine $t_a \ll t_e$; not, however, so

 

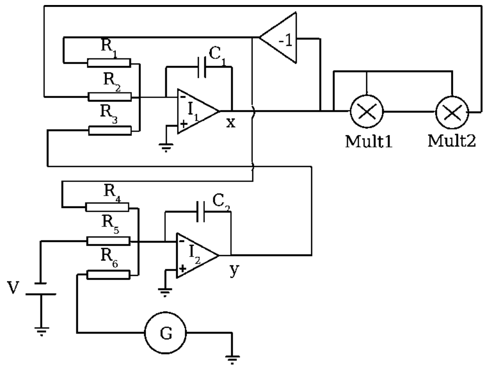


FIG. 1. Sketch of the electronic circuit. $I_1, I_2$, integrators; $R_1, R_2, R_3, R_4, R_5, R_6$, resistors; $C_1, C_2$, capacitors; $\mathrm{Mult}_1, \mathrm{Mult}_2$, multipliers; $G$, noise generator; $V$, bias generator.

large as to prevent the fluctuations of the excursion time from becoming relevant.

The used apparatus is schematically reproduced in Fig. 1. It consists of an electronic analog simulator implemented using common semiconductor devices. The architecture of the circuit has been implemented to simulate the FitzHugh-Nagumo excitable system [18]. The model equations governing the circuit dynamics are therefore

$$\dot{x} = c\left(x - \frac{1}{3}x^3 - y\right),$$

$$\dot{y} = ex + a|V| - b\xi, \tag{2}$$

where the variables $x$ and $y$ are the output voltages from the first and second integrators $I_1$ and $I_2$, respectively, $\xi$ represents an external noise voltage applied by means of a variable generator $G$, and the parameters $a, b, c, e$ are linked to the values of resistors and capacitors in the circuit by the relations $a=1/R_5C_2$, $b=1/R_6C_2$, $c=1/R_1C_1$, and $e=1/R_4C_2$. While $C_1$ will be used as a variable capacitor, the other values of resistors and capacitors are $C_2=33$ nF, $R_1=R_3=R_4=100$ k$\Omega$, $R_2=300$ k$\Omega$, $R_5=90$ k$\Omega$, and $R_6=125$ k$\Omega$. The integrators $I_1$ and $I_2$ have been implemented using linear technology LT1114CN four-quadrant operational amplifiers, while the multipliers $\mathrm{Mult}_1$ and $\mathrm{Mult}_2$ are MLT04 analog devices. The acquisition of the experimental data has been performed by means of a LeCroy digital oscilloscope and by means of a real time acquisition board connected to a personal computer provided with LABVIEW software.

Noise is applied to the circuit by means of the variable generator $G$. In order to produce a class of correlated noises, we started by analogically acquiring a long time series of Gaussian white noise. The signal is then digitally filtered with a high bandpass cutoff positioned at $f=40$ Hz. This is done to avoid low frequency forcings, which have the effect of driving the system from the excitable to the self-oscillatory regime. The Fourier spectrum of the filtered signal is then manipulated to impose an amplitude distribution scaling as $1/f^{\alpha}$ (with $0 \leqslant \alpha \leqslant 2$ being a tunable parameter) in the remaining frequency range. In the very same spirit of the surrogate signal technique, the phases of each Fourier component are further randomized, and an inverse Fourier transform of the signal is realized. The obtained digital signal is passed through an analog converter driving the variable generator $G$. By the Wiener-Khintchine theorem [19], the autocorrelation function of the noisy source (the inverse Fourier transform of the noise power spectrum) is divergent in all the range $0 \leqslant \alpha \leqslant 2$. This is at variance with previous studies with Ornstein-Uhlenbeck processes [14,15], where the Gaussian features in the noise power spectra induced a Gaussian scaling in the correlation function, thus allowing for introducing a characteristic time scale for the noise correlation properties.

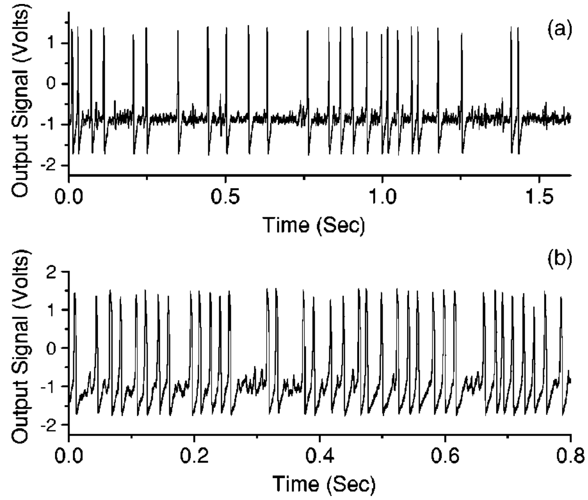


FIG. 2. Output voltage of the circuit ($x$ variable) when subjected to $1/f$ noise ($\alpha=1$), for two different values of the rms noise level: (a) low noise, no coherence; (b) intermediate noise, high coherence.

When such a noise source is added to the circuit, a typical time series for the output variable $x$ consists of a sequence of spikes on top of a noisy background (see Fig. 2). By carefully selecting a threshold (whose value must be such that the noisy fluctuations close to the fixed point are washed out), and by monitoring the instants at which the output signal overcomes such threshold with a positive derivative, one can reconstruct the corresponding $I_{\mathrm{IS}}$ sequence, whose distribution allows us to calculate the coherence factor according to Eq. (1). Furthermore, in order to reduce the measurement errors, in all cases the experiment has been run in several replicas, and the obtained values of $C$ have been averaged out.

We start now by considering the case of white noise ($\alpha=0$), and by reporting a series of experimental observations and characterizations of CR in our system, obtained by modulating both the distance to threshold and the characteristic time scales of the dynamics. The distance to threshold can be operationally adjusted by modulating the bias generator $V$. In the absence of noise, the circuit begins to behave as a self-sustained oscillator at $V_c=-756$ mV, while for $V < V_c$ the system is in an excitable state. As for the time scale of the dynamics, this is accounted for by the parameter $c$ in Eq. (2), and it can be experimentally adjusted by changing the circuit capacitor $C_1$.

We start from investigating the system response to noise when the bias voltage parameter is modulated. To this pur-

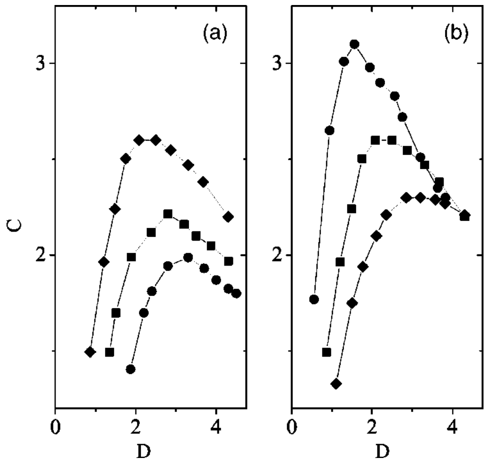


FIG. 3. Coherence resonance effects of a forcing white noise. (a) Coherence factor $C$ as a function of the rms noise amplitude $D$ for $C_1 = 3$ nF and $V = -800$ mV (curve with diamonds), $V = -900$ mV (curve with squares), and $V = -1{,}000$ mV (curve with circles). (b) Coherence factor $C$ as a function of the rms noise amplitude $D$ for $V = -800$ mV and $C_1 = 1$ nF (curve with circles), $C_1 = 3$ nF (curve with squares), and $C_1 = 6$ nF (curve with diamonds). In all cases, data refer to ensemble averages over five different realizations of the noisy forcing.

pose, we measure the coherence factor $C$ as a function of the noise intensity $D$ for various values of $V$. Results are reported in Fig. 3(a).

CR is found, which is significantly enhanced as the bias voltage increases to approach the excitability threshold $V_c = -756$ mV. Precisely, Fig. 3(a) shows that, as we increase the bias voltage, the resonant behavior can occur for lower values of the noise intensity $D$ and gives rise to higher resonance peaks, meaning that CR becomes more evident and enlarged. It is worth mentioning that similar resonant effects at different excitation thresholds were reported experimentally in Ref. [11].

Another relevant parameter of system (2) is $c$, governing the ratio in the characteristic time scales $t_a$ and $t_e$. Precisely, if $c \gg 1$, the trajectory moves very rapidly along the $x$ axis in phase space, while excursions along the $y$ axis are relatively slow. Making $c \sim 1$ implies that the two excursions take place with similar time scales, and this in its turn yields a deterioration of the resonant behavior.

A modulation of the $c$ parameter in our experiment can be achieved by gradually changing the value of the capacitor $C_1$. This is because $c$ has an inverse relation with the capacitance of $C_1$, and therefore higher values of $C_1$ provide lower values for the $c$ parameter. By changing $C_1$ we can then properly modulate the time spent by the trajectory during the spiking orbit.

In Fig. 3(b) we report the coherence factor $C$ as a function of the rms noise amplitude for various values of $C_1$. As expected, larger and larger values of $C_1$ [lower and lower values of the parameter $c$ in Eq. (2)] make the two time scales of the dynamics closer and closer to each other, thus inducing a progressive deterioration of the resonant behavior.

The very same qualitative behavior has been observed for the whole class of colored noise produced with the procedure

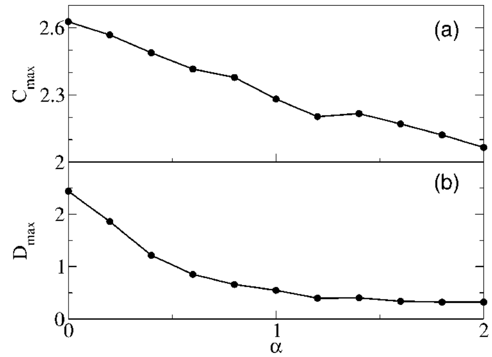


FIG. 4. Value [(a) $C_{\max}$] and position [(b) $D_{\max}$] of the coherence resonant peak vs the parameter $\alpha$, for $V = -800$ mV and $C_1 = 3$ nF.

described above. In order to quantitatively assess the effects of increasing noise correlation (increasing values of $\alpha$) on the system, we fixed the system parameters at $V = -800$ mV and $C_1 = 3$ nF, and we performed a series of CR measurements for various values of the parameter $\alpha$. In all cases a well shaped resonant curve has been observed in the plot $C$ vs $D$. For each measurement, the maximum coherence $C_{\max}$ reached by the system at the resonant point $D_{\max}$ has been recorded. In Fig. 4, both $C_{\max}$ [Fig. 4(a)] and $D_{\max}$ [Fig. 4(b)] are shown as functions of the parameter $\alpha$. The resonant effect is progressively deteriorated, as correlation in the noisy source increases. This is reflected by the almost linear decrease in $C_{\max}$ shown in Fig. 4(a), which is, however, accompanied by a monotonic decrease in $D_{\max}$ [visible in Fig. 4(b)], witnessing that noise correlations actually produce resonant behaviors for lower forcing noise amplitudes.

It is known that the phenomenon of coherence resonance can be induced also by properly tuning parameters other than the noise intensity. This was proven numerically by studying coherence resonance as a function of the noise correlation time for an Ornstein-Uhlenbeck noise process [20]. In our case, we performed a series of measurements in which the noise intensity $D$ was fixed and the correlation parameter $\alpha$ was varied. The results are shown in Fig. 5, where the co-

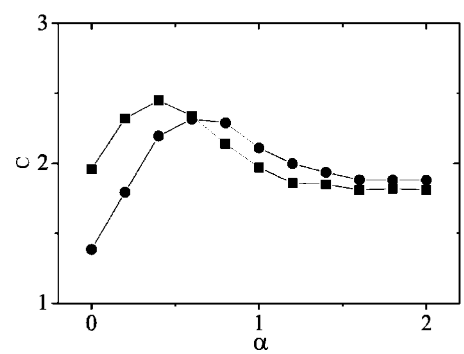


FIG. 5. Coherence resonance effects due to a forcing colored noise. Coherence factor $C$ as a function of the parameter $\alpha$ for $D = 1.2$ (curve with squares) and 0.8 (curve with circles). Other parameters as in the caption of Fig. 4. In all cases, data refer to ensemble averages over five different realizations of the noisy forcing.

herence factor $C$ is reported as a function of $\alpha$ for $D=1.2$ (curve with squares) and 0.8 (curve with circles). For a comparison with the results of Fig. 4, in both cases the other parameters have been kept at $V=-800$ mV and $C_1=3$ nF. From Fig. 5 it is evident that coherence resonance is induced in our system also when properly tuning the correlation properties of the forcing noisy source.

In conclusion, we have given evidence of coherence resonance in an excitable electronic circuit whose dynamics obeys the FitzHugh–Nagumo model system, under the application of a class of colored noise sources, whose spectral distributions scale as $1/f^{\alpha}$. The resonance behavior appears to be a generic feature of excitable systems under modulation of the main dynamical parameters, and regardless of the specific properties of the stochastic process acting on the system. In particular, coherence resonance can be significantly reinforced when experimental parameters are tuned in order to place the stable fixed point closer to the excitability threshold of spiking behavior, or when the time scales of the circuit are properly adjusted. We have furthermore shown that increasing noise correlations produce a twofold effect: from one side they progressively deteriorate the quality of coherence resonance; from the other side they anticipate the resonant effect, insofar as they yield a resonant peak occurring at lower forcing noise amplitudes.

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